① $y = \cos(y-x)$ find y'

$$y' = -\sin(y-x)(y'-1)$$

$$y' = -y'\sin(y-x) + \sin(y-x)$$

$$+y'\sin(y-x) \quad +y'\cancel{\sin(y-x)}$$

$$y' + y'\sin(y-x) = \sin(y-x)$$

$$\frac{y'(1+\sin(y-x))}{1+\sin(y-x)} = \frac{\sin(y-x)}{1+\sin(y-x)}$$

$$\boxed{y' = \frac{\sin(y-x)}{1+\sin(y-x)}}$$

② $\dfrac{1-\csc^2 x}{1-\sin^2 x}$ Show how it equals $-\csc^2 x$

$$\frac{-\cot^2 x}{\cos^2 x} = \frac{-\cos^2 x}{\sin^2 x} \div \frac{\cos^2 x}{1} = \frac{-\cancel{\cos^2 x}}{\sin^2 x} \cdot \frac{1}{\cancel{\cos^2 x}} = \frac{-1}{\sin^2 x} = \boxed{-\csc^2 x}$$

③ Find where there is a horizontal tangent line. $f(x) = x^2 - 8x + 2$

$$f'(x) = 2x - 8$$

$$2x - 8 = 0$$
$$+8 \quad +8$$
$$2x = 8$$
$$x = 4$$

$$f(4) = (4)^2 - 8(4) + 2$$
$$16 - 32 + 2$$
$$f(4) = -14$$

$$\boxed{(4, -14)}$$

④ $y = \dfrac{x}{x+4}$ Find $\dfrac{d^2y}{dx^2}$

$$y' = \frac{(1)(x+4) - (x)(1)}{(x+4)^2} = \frac{\cancel{x}+4-\cancel{x}}{(x+4)^2} = \frac{4}{(x+4)^2} = 4(x+4)^{-2}$$

$$y'' = -8(x+4)^{-3}(1) = \boxed{\frac{-8}{(x+4)^3}}$$

⑤ $y = (2-x^2)^3$ Find $y''(-1)$

$$y' = 3(2-x^2)^2(-2x) = -6x(2-x^2)^2$$

$$y'' = (-6)(2-x^2)^2 + (-6x)(2(2-x^2)^1(-2x))$$

$$y'' = -6(2-x^2)^2 + 24x^2(2-x^2)$$

$$y''(-1) = -6(2-(-1)^2)^2 + 24(-1)^2(2-(-1)^2)$$

$$y''(-1) = -6(1) + 24(1) = \boxed{18}$$

⑥ $h(x) = \dfrac{2}{\sqrt[4]{x^2-x}}$ Find $h'(x)$

$$h(x) = 2(x^2-x)^{-1/4}$$

$$h'(x) = 2\left(-\frac{1}{4}(x^2-x)^{-5/4}(2x-1)\right)$$

$$h'(x) = \frac{-1(2x-1)}{2(x^2-x)^{5/4}} = \boxed{\frac{1-2x}{2(x^2-x)^{5/4}}}$$

⑦ Find an equation of a tangent line where $x=2$ of $f(x) = -x^2+8x-1$ (Write it in $y=mx+b$ form)

$f(2) = -(2)^2+8(2)-1$

$f(2) = -4+16-1$

$f(2) = 11$

$(2,11)$

$f'(x) = -2x+8$

$f'(2) = -2(2)+8$

$f'(2) = -4+8 = 4$

$f'(2) = 4$

$y-11 = 4(x-2)$

$y-11 = 4x-8$

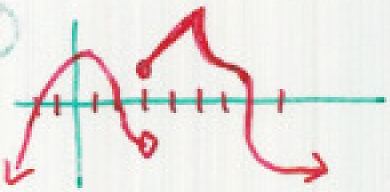
$+11 \quad +11$

$\boxed{y=4x+3}$

① $S(t) = 4t^3 - 5t^2 + 1$ $\quad a(5) = ?$

$S'(t) = 12t^2 - 10t$ $\quad S''(t) = a(t)$ $\quad a(5) = 24(5) - 10$

$S''(t) = 24t - 10$ $\quad 120 - 10 = 110$

② 

Where not differentiable??

$x = 3, 5, 7$

③ $f(x) = -2x^2 - 3x$ $\quad$ Derive by limit process (set up)

$$\lim_{\Delta x \to 0} \frac{[-2(x+\Delta x)^2 - 3(x+\Delta x)] - [-2x^2 - 3x]}{\Delta x}$$

④ $f(x) = 3x^2 - 5x + 1$ $\quad$ Where does it have a slope of 7?

$f'(x) = 6x - 5$ $\quad 6x - 5 = 7$, $\quad +5 \quad +5$, $\quad \frac{6x}{6} = \frac{12}{6}$ $\quad x = 2$

$f(2) = 3(2)^2 - 5(2) + 1$

$f(2) = 12 - 10 + 1 = 3$ $\quad (2, 3)$

⑤ $3x^3 - 27x = 0$ $\quad$ solve for x

$3x(x^2 - 9) = 0$ $\quad x = 0, 3, -3$

$3x(x-3)(x+3) = 0$

⑥ $y = \frac{x}{2x-3}$ $\quad \frac{d^2y}{dx^2} = ?$

$$y' = \frac{1(2x-3) - x(2)}{(2x-3)^2} = \frac{-3}{(2x-3)^2}$$

$$y'' = \frac{0(2x-3)^2 - (-3(2(2x-3)(2)))}{(2x-3)^4} = \frac{12(2x-3)}{(2x-3)^4}$$

$$y'' = \frac{12}{(2x-3)^3}$$

⑦ $y = \frac{1}{\sqrt{x^2-2}}$ $\quad y' = ?$

$y = (x^2-2)^{-\frac{1}{2}}$

$y' = -\frac{1}{2}(x^2-2)^{-\frac{3}{2}}(2x)$

$$y' = \frac{-x}{(x^2-2)^{\frac{3}{2}}}$$

(8) $f(x)= \sec^5 6x$ $\quad f'(x)=?$

$f(x)=(\sec 6x)^5$ $\quad f'(x)=5(\sec 6x)^4(\sec 6x \tan 6x)(6)$

$\boxed{f'(x)= 30\sec^5 6x \tan 6x}$

(9) $y=\cos^2 x-\sin^2 x$ $\quad y'=?$

$\sin^2 x = 1-\cos^2 x$ $\quad y=\cos^2 x-(1-\cos^2 x)$ $\quad y=2\cos^2 x-1$

$y'=4(\cos x)(-\sin x)$ $\quad \boxed{y'=-4\cos x \sin x}$ $\quad 2\cos x \sin x=\sin 2x$ $\quad \boxed{y'=-2\sin 2x}$

(10) $x=\cot(xy)$ $\quad \frac{dy}{dx}=?$

$\frac{1}{-\csc^2 xy}=\frac{-\csc^2(xy)(1y+xy')}{-\csc^2(xy)}$ $\quad -\sin^2(xy)=y+xy'$

$\quad -y \quad -y$

$\frac{-1(\sin^2(xy)+y)}{x}=\frac{xy'}{x}$ $\quad \boxed{y'=\frac{-(\sin^2(xy)+y)}{x}}$

(11) $y=5x^3-2x$ $\quad$ y is decreasing at 5 units per second
When $x=2$ what is the rate of x?

$\frac{dy}{dt}=(15x^2-2)\frac{dx}{dt}$ $\quad \frac{dy}{dt}=-5$ $\quad x=2$ $\quad -5=(15(2)^2-2)\frac{dx}{dt}$ $\quad \frac{-5}{58}=\frac{58\frac{dx}{dt}}{58}$

$\boxed{\frac{dx}{dt}=-\frac{5}{58} \text{ units/sec}}$

(12) 

$\frac{dA}{dt}=5\frac{in^2}{sec}$ $\quad r=8$ $\quad \frac{dr}{dt}=?$

$A=\pi r^2$ $\quad \frac{dA}{dt}=2\pi r\cdot\frac{dr}{dt}$ $\quad 5=2\pi(8)\frac{dr}{dt}$ $\quad \frac{5}{16\pi}=\frac{16\pi\frac{dr}{dt}}{16\pi}$

$\boxed{\frac{dr}{dt}=\frac{5}{16\pi}\frac{in}{sec}}$

① $f(x)=\dfrac{2}{(x-x^2)^3}$ Find the equation for the tangent line at $x=2$.

$f(x)=2(x-x^2)^{-3}$

$f'(x)=-6(x-x^2)^{-4}(1-2x)$

$f'(x)=\dfrac{-6(1-2x)}{(x-x^2)^4}$

$\boxed{f(2)=\dfrac{2}{(2-(2)^2)^3}=\dfrac{2}{-8}=-\frac{1}{4}\ (2,-\tfrac{1}{4})}$

$\boxed{f'(2)=\dfrac{-6(1-2(2))}{((2)-(2)^2)^4}=\dfrac{-6(-3)}{16}}$

$\boxed{f'(2)=\dfrac{18}{16}=\dfrac{9}{8}}$

$y+\frac{1}{4}=\frac{9}{8}(x-2)$

$y+\frac{1}{4}=\frac{9}{8}x-\frac{9}{4}$

$-\frac{1}{4}\qquad -\frac{1}{4}$

$-\frac{9}{4}-\frac{1}{4}=-\frac{10}{4}=-\frac{5}{2}$

$\boxed{y=\frac{9}{8}x-\frac{5}{2}}$

---

② $3xy=\sec(x-y)$ Find $\dfrac{dy}{dx}$

$(3)(y)+(3x)(y')=(\sec(x-y)\tan(x-y))(1-y')$

$3y+3xy'=\sec(x-y)\tan(x-y)-y'\sec(x-y)\tan(x-y)$

$3xy'+y'\sec(x-y)\tan(x-y)=\sec(x-y)\tan(x-y)-3y$

$\dfrac{y'(3x+\sec(x-y)\tan(x-y))}{3x+\sec(x-y)\tan(x-y)}=\dfrac{\sec(x-y)\tan(x-y)-3y}{3x+\sec(x-y)\tan(x-y)}$

$$\boxed{y'=\dfrac{\sec(x-y)\tan(x-y)-3y}{3x+\sec(x-y)\tan(x-y)}}$$

③ $y = \sqrt[3]{\tan(7x)}$ Find y'

$y = (\tan(7x))^{1/3}$

$y' = \frac{1}{3}(\tan(7x))^{-2/3}(\sec^2(7x))(7)$

$$\boxed{y' = \frac{7\sec^2(7x)}{3\sqrt[3]{\tan^2(7x)}}}$$

---

④ $f(x) = \cot^2 x - \sec^2 x$ Find $f'(x)$

$f(x) = (\cot x)^2 - (\sec x)^2$

$f'(x) = 2(\cot x)(-\csc^2 x) - 2(\sec x)(\sec x \tan x)$

$$\boxed{f'(x) = -2(\csc^2 x \cot x + \sec^2 x \tan x)}$$

---

⑤ $f(x) = \dfrac{x^2+3}{x}$ Find $f'(5)$

$f(x) = x + \frac{3}{x} = x + 3x^{-1}$

$f'(x) = 1 - 3x^{-2} = 1 - \frac{3}{x^2}$

$f'(5) = 1 - \frac{3}{25} = \frac{22}{25}$

⑥ $y = \dfrac{5-x}{x^2-4}$ Find $\dfrac{d^2y}{dx^2}$

$$y' = \frac{(-1)(x^2-4)-(5-x)(2x)}{(x^2-4)^2} = \frac{-x^2+4-10x+2x^2}{(x^2-4)^2} = \frac{x^2-10x+4}{(x^2-4)^2}$$

$$y'' = \frac{(2x-10)(x^2-4)^2-(x^2-10x+4)(2(x^2-4)2x)}{(x^2-4)^4} = \frac{2\cancel{(x^2-4)}\left((x-5)(x^2-4)-(2x^3-20x^2+8x)\right)}{(x^2-4)^{\cancel{4}\,3}}$$

$$\frac{2(x^3-4x-5x^2+20-2x^3+20x^2-8x)}{(x^2-4)^3}$$

$$\frac{2(-x^3+15x^2-12x+20)}{(x^2-4)^3}$$

⑦ Rewrite $\dfrac{\csc x}{1+\cot^2 x}$ in terms of $\sin x$ & $\cos x$.

$$\frac{\csc x}{\csc^2 x} = \frac{1}{\csc x} = \boxed{\sin x}$$