Chapter 4

Integration

Indefinite Integral

or

Antiderivative

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$$

$$\int \left(4x^3 - 2x\right) dx = x^4 - x^2 + c$$

Find the Particular Solution

or

Solve the Differential Equation

$$\frac{dy}{dx} = 4x - 1,\ at\ (1,5)$$

$$y = 2x^2 - x + 4$$

1. Change into a differential equation.
2. Integrate both sides of the equation.
3. Find c by plugging in the coordinate.
4. Replace c and write the particular solution.

1st Fundamental Theorem of Calculus

Definite Integral

or

Area Under the Curve
on the interval [a,b]

$$\int_a^b f(x)dx = F(b) - F(a)$$

$$\int_1^3 (2x-2)\,dx = \left((3)^2 - 2(3)\right) - \left((1)^2 - 2(1)\right) = 4$$

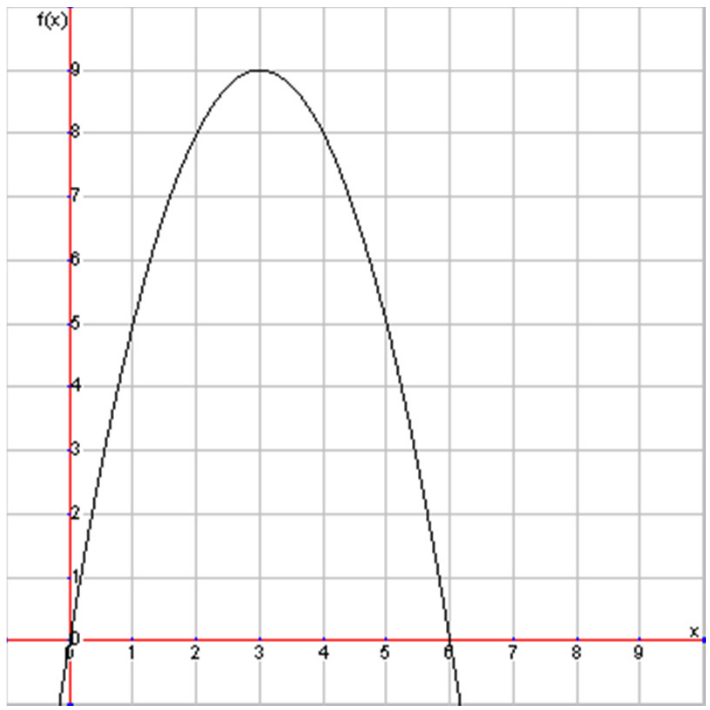
Area below the x-axis is

NEGATIVE

Approximate the Area Under a Curve Using a Left-Sided Sum

$$f(x) = -x^2 + 6x \text{ on } [1,4] \text{ with } 3 \text{ subintervals}$$

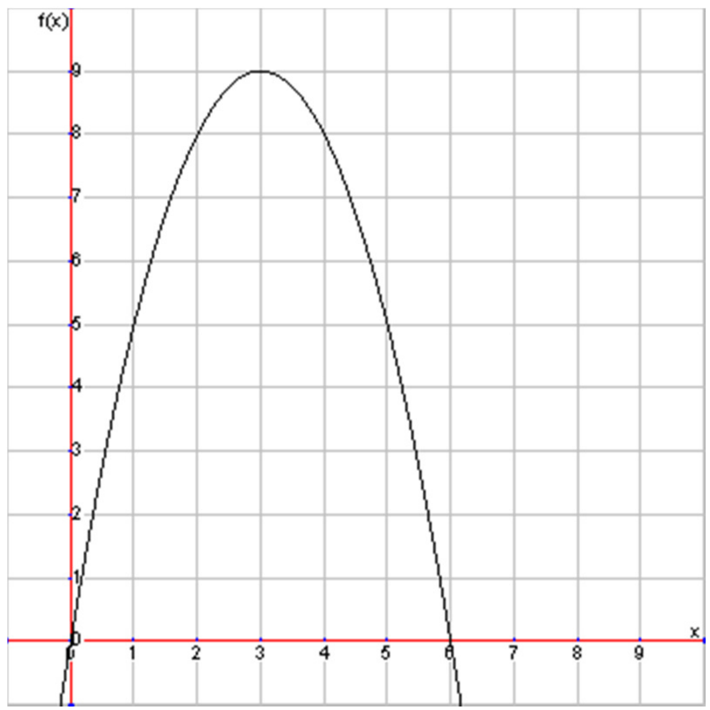
$$A \approx \frac{4-1}{3}\left[f(1) + f(2) + f(3)\right]$$



Approximate the Area Under a Curve Using a Right-Sided Sum

$$f(x) = -x^2 + 6x \text{ on } [1,4] \text{ with } 3 \text{ subintervals}$$

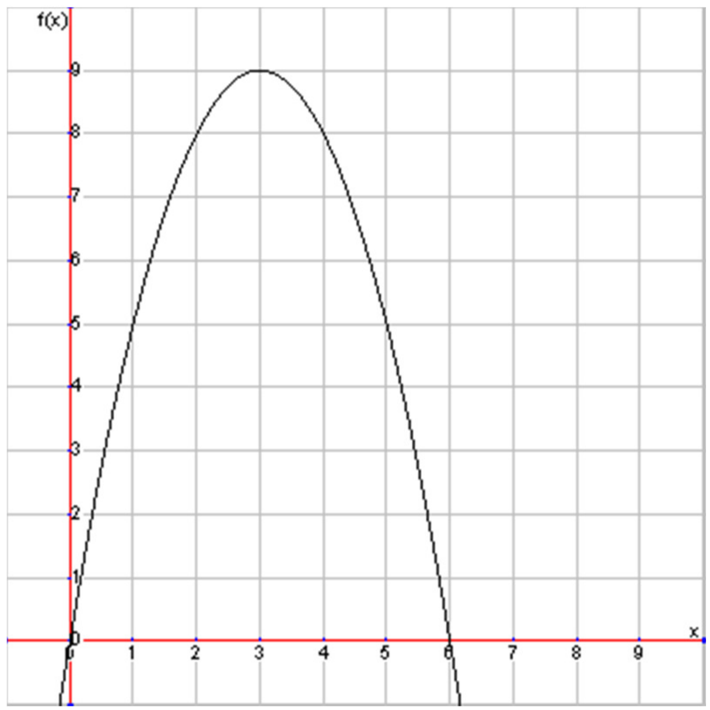
$$A \approx \frac{4-1}{3}\left[f(2) + f(3) + f(4)\right]$$



Approximate the Area Under a Curve Using a Midpoint Sum

$$f(x) = -x^2 + 6x \text{ on } [1,4] \text{ with 3 subintervals}$$

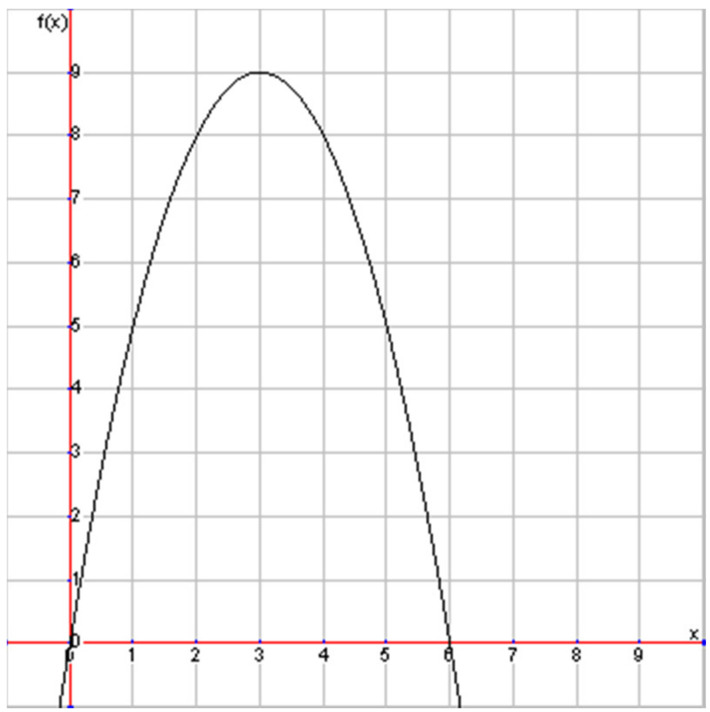
$$A \approx \frac{4-1}{3}\left[f(1.5) + f(2.5) + f(3.5)\right]$$



Approximate the Area Under a Curve Using a Trapezoid Sum

$$f(x) = -x^2 + 6x \text{ on } [1,4] \text{ with } 3 \text{ subintervals}$$

$$A \approx \frac{4-1}{3} \bullet \frac{1}{2}\left[f(1) + 2f(2) + 2f(3) + f(4) \right]$$



Mean Value Theorem (MVT)

or

Average Value

Mean Value Theorem

$$f(c)(b-a)=\int_a^b f(x)dx$$

Average Value

$$f(c)=\frac{1}{b-a}\int_a^b f(x)dx$$

Find the x value where you get the Average Value

$$f(x) = 2x - 1, \ [2, 4]$$

$$2x - 1 = \frac{1}{4-2}\int_2^4 (2x-1)\,dx$$

1. Find the Average Value.
2. Set the original function equal to the Average Value.
3. Solve for x.

2^{nd} Fundamental Theorem of Calculus

$$\frac{d}{du}\left[\int_a^u f(t)\,dt\right] = f(u)\bullet u'$$

$$\frac{d}{dx}\left[\int_1^{4x^3} t^2\,dt\right] = \left(4x^3\right)^2\left(12x^2\right) = 192x^8$$

$$\int_a^a f(x)\,dx =$$

0

$$\int_b^a f(x)\,dx =$$

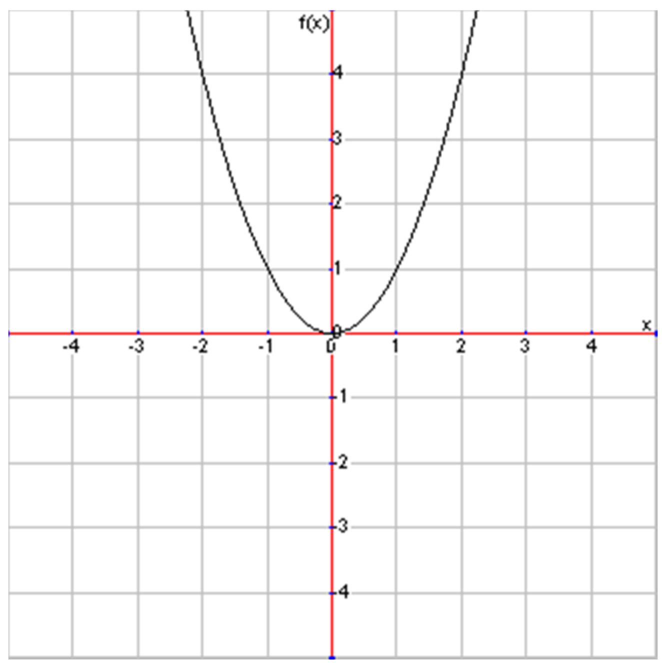
$$-\int_a^b f(x)\,dx$$

$$\int_a^b \left(f(x) + k \right) dx =$$

$$\int_a^b f(x)\,dx + \int_a^b k\,dx$$

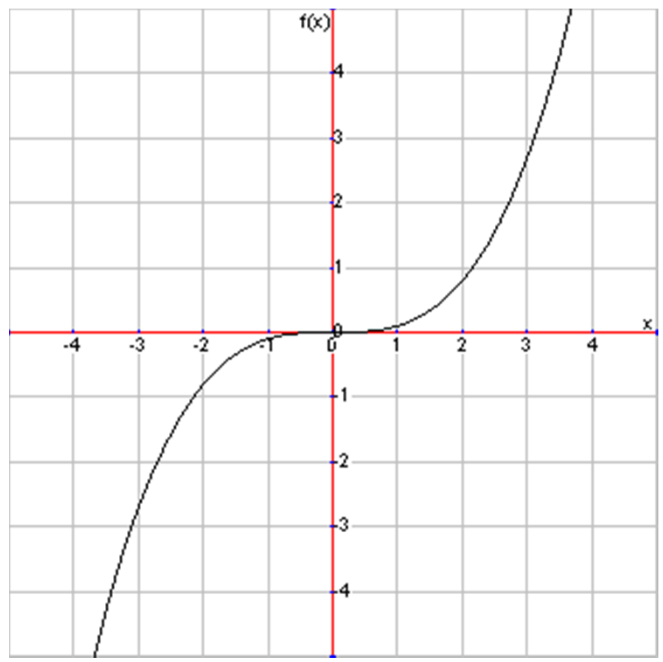
Integrate an Even Function

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$



Integrate an Odd Function

$$\int_{-a}^{a} f(x)\,dx = 0$$



U-Substitution

or

Change of Variables

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du = F(u) + C$$

$$\int 2x\left(x^2 - 3\right)^3 dx = \int (u)^3\,du$$

$$u = x^2 - 3 \text{ and } du = 2x\,dx$$

Find Definite Integral Using

U-Substitution

or

Change of Variables

$$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

$$\int_0^2 2x\left(x^2-3\right)^3 dx = \int_{g(0)}^{g(2)} (u)^3\,du = \int_{-3}^{1} (u)^3\,du$$

$$u = x^2 - 3 \quad and \quad du = 2x\,dx$$