# Implicit Differentiation

# Find the derivative

$$x^4 - 3xy^2 = 8$$

$$4x^3 - \left((3)(y^2) + (3x)(2y \cdot y')\right) = 0$$

$$\cancel{4x^3} - \cancel{3y^2} - 6xyy' = 0 \quad -4x^3 + 3y^2$$

$$-4x^3 \quad +3y^2$$

$$\frac{-\cancel{6}xyy'}{-6x\cancel{y}} = \frac{-4x^3 + 3y^2}{-6xy}$$

$$\boxed{y' = \frac{-4x^3 + 3y^2}{-6xy}}$$

# Find the derivative

$5\tan y = y^2 - 3x^4 + 4$

$5\sec^2 y \cdot y' = 2y \cdot y' - 12x^3$

$-2yy' \qquad -2yy'$

$5\sec^2 y\, y' - 2yy' = -12x^3$

$$\frac{y'(\cancel{5\sec^2 y - 2y})}{\cancel{5\sec^2 y - 2y}} = \frac{-12x^3}{5\sec^2 y - 2y}$$

$$\boxed{y' = \frac{-12x^3}{5\sec^2 y - 2y}}$$

# Find the derivative

$$x^3 \sec y = 2 - 3x^4 y$$

$$(3x^2)(\sec y) + (x^3)(\sec y \tan y\, y') = (-12x^3)(y) + (-3x^4)(y')$$

$$x^3 \sec y \tan y\, y' + 3x^4 y' = -12x^3 y - 3x^2 \sec y$$

$$\frac{y'(x^3 \sec y \tan y + 3x^4)}{x^3 \sec y \tan y + 3x^4} = \frac{-3x^2(4xy + \sec y)}{x^3 \sec y \tan y + 3x^4}$$

$$y' = \frac{-3x^2(4xy + \sec y)}{x^3(\sec y \tan y + 3x)}$$

$$\boxed{y' = \frac{-3(4xy + \sec y)}{x(\sec y \tan y + 3x)}}$$

# Find the derivative at (1,-2)

$x \quad y$

$$2x^2 + 3y^2 = 14$$

$$\cancel{4x} + 6y \cdot y' = 0$$

$$\cancel{-4x} \qquad -4x$$

$$\frac{\cancel{6y}y'}{\cancel{6y}} = \frac{-4x}{6y}$$

$$y' = -\frac{2x}{3y}$$

$$y' = \frac{-2(1)}{3(-2)} = \frac{-2}{-6} = \frac{1}{3}$$

$$4(1) + 6(-2)y' = 0$$

$$\cancel{4} + -12y' = 0$$

$$\cancel{-4} \qquad -4$$

$$\frac{\cancel{-12}y'}{\cancel{-12}} = \frac{-4}{-12}$$

$$y' = \frac{1}{3}$$

# Find the second derivative

$2\sin x - y^3 = 1$

$$\cancel{2\cos x} - 3y^2\cdot y' = 0$$

$$\cancel{-2\cos x} \qquad\qquad -2\cos x$$

$$\frac{\cancel{-3}y^2 y'}{\cancel{-3}y^2} = \frac{-2\cos x}{-3y^2}$$

$$y' = \frac{2\cos x}{3y^2}$$

$$y'' = \frac{(-2\sin x)(3y^2) - (2\cos x)(6y\cdot y')}{(3y^2)^2}$$

$$y'' = \frac{-6y^2\sin x - 12y\cos x\left(\frac{2\cos x}{3y^2}\right)}{9y^4} = \frac{\frac{-6y^2\sin x\cdot 3y^2}{3y^2} - \frac{24y\cos^2 x}{3y^2}}{9y^4}$$

$$y'' = \frac{-18y^4\sin x - 24y\cos^2 x}{3y^2(9y^4)}$$

$$y'' = \frac{-6y(3y^3\sin x + 4\cos^2 x)}{27y^6}$$

$$\boxed{y'' = \frac{-2(3y^3\sin x + 4\cos^2 x)}{9y^5}}$$