$$f(x)=\frac{x^2-25}{x^2+3x-10}$$ Describe all discontinuities as good as possible.

$$f(x)=\frac{(x-5)\cancel{(x+5)}}{\cancel{(x+5)}(x-2)}$$

$x=-5$ is a hole (removable discontinuity)

$$\lim_{x\to -5} f(x)=\frac{-5-5}{-5-2}=\frac{-10}{-7}=\frac{10}{7}$$

$$f(x)=\frac{x-5}{x-2}$$

$x=2$ is a vertical asymptote
nonremovable discontinuity

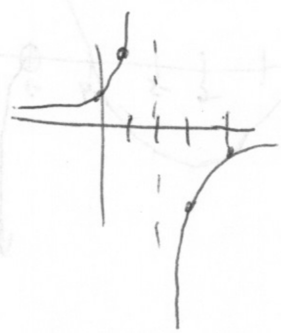
$$\frac{2-5}{2-2}=\frac{-3}{0}$$

$$\lim_{x\to 2^-} f(x)=\infty$$

$$\lim_{x\to 2} f(x)=\text{DNE}$$

$$\lim_{x\to 2^+} f(x)=-\infty$$

0	1	$\frac{3}{2}$	2	$2\frac{1}{2}$	3	4
$\frac{-5}{-2}$	4				$\frac{-2}{1}$	$\frac{-1}{2}$



$$\lim_{x\to 0} \frac{4\sin(7x)}{2\cos(x)-2}-\frac{x-8}{2}$$

$$\lim_{x\to 0} \frac{7x\,(4\sin(7x))}{7x(-2)(1-\cos x)}-\frac{x-8}{2}$$

$$\lim_{x\to 0} \left(\frac{x}{1-\cos x}\right)\left(\frac{\sin(7x)}{7x}\right)\left(\frac{28}{-2}\right)-\frac{x-8}{2}$$

$$(0)\quad (1)\quad (-14)\quad -\frac{-8}{2}=0+4=4$$

which
Guarantees a zero. $[-3, 5]$ Where?

a) $f(x) = \frac{3x}{x+1}$ b) $g(x) = -x^4 - 1$ c) $\frac{x^2-16}{x+4} = h(x)$

a) Discontinuity at $x = -1$

No IVT

b) Continuous $(-\infty, \infty)$

$g(-3) = -(81) - 1 = -82$

$g(5) = -(5)^4 - 1 = -626$

No IVT

c) $x = -4$ is discontinuous but outside the interval

$h(x) = x - 4$

$h(-3) = -3 - 4 = -7$

$h(5) = 5 - 4 = 1$

IVT ok

$x - 4 = 0$

$x = 4$ is your zero

$$f(x) = \begin{cases} x^2 - 5, & x < 1 \\ 3x - c, & x \geq 1 \end{cases}$$

Find c so that it is continuous.

$1^2 - 5 = 3(1) - c$

$-4 = 3 - c$

$-3 \quad -3$

$-7 = -c$

$c = 7$