Continuity and One-Sided Limits

Where is each function discontinuous and what type of discontinuity is it?

$$g(x) = \frac{4x^2 - 25}{10 + 4x}$$

$\cancel{10} + 4x = 0$

$\cancel{-10} \qquad -10$

$\frac{\cancel{4}x}{\cancel{4}} = \frac{-10}{4}$

$\boxed{x = -\frac{5}{2}}$

Removable Discontinuity

$$g(x) = \frac{(2x-5)\cancel{(2x+5)}}{2\cancel{(2x+5)}}$$

$$g(x) = \frac{2x-5}{2}$$

$$h(x) = \frac{x^2 - 16}{x - 2}$$

$x - 2 = 0 \qquad \boxed{x = 2}$

Nonremovable dis.

$$h(x) = \frac{(x-4)(x+4)}{x-2}$$

$$f(x) = \frac{|x+2|}{x+2}$$

$x + 2 = 0 \qquad \boxed{x = -2}$

Non removable dis.

$\lim_{x \to -2^-} f(x) \qquad \lim_{x \to 2^+} f(x)$

-4	-3	-2.5	-2	-1.5	-1	0
-1	-1	-1	?	1	1	1

$$\frac{|-4+2|}{-4+2} = \frac{2}{-2} = -1$$

$$\frac{|-3+2|}{-3+2} = \frac{1}{-1} = -1$$

$$\frac{|0+2|}{0+2} = \frac{2}{2} = 1 \qquad \frac{|-1+2|}{-1+2} = \frac{1}{1} = 1$$

$$f(x)=\begin{cases}3x-5, & x<-2\\ \dfrac{2-x}{x}, & x\geq -2\end{cases}$$

$a)\ f(-2)=$ $\dfrac{2--2}{-2}=\dfrac{4}{-2}=\boxed{-2}$

$b)\ \lim\limits_{x\to -2^-} f(x)=$ $3(-2)-5=-6-5=\boxed{-11}$

$c)\ \lim\limits_{x\to -2^+} f(x)=$ $\boxed{-2}$

$d)\ \lim\limits_{x\to -2} f(x)=$ $\boxed{\text{DNE}}$

$e)$ Where is f(x) discontinuous?

$\boxed{X=-2\ \&\ 0}$

Find the value of a so that $f(x)$ is continuous

$$f(x) = \begin{cases} ax - 1, & x \le 2 \\ 4 - x, & x > 2 \end{cases}$$

$ax - 1 = 4 - x$

$a(2) - 1 = 4 - 2$

$2a \cancel{-1} = 2$

$\quad\ \cancel{+1} \quad +1$

$\frac{\cancel{2}a}{\cancel{2}} = \frac{3}{2}$

$a = \frac{3}{2}$

Verify that the Intermediate Value Theorem applies on the interval, and find the value of c guaranteed by the theorem.

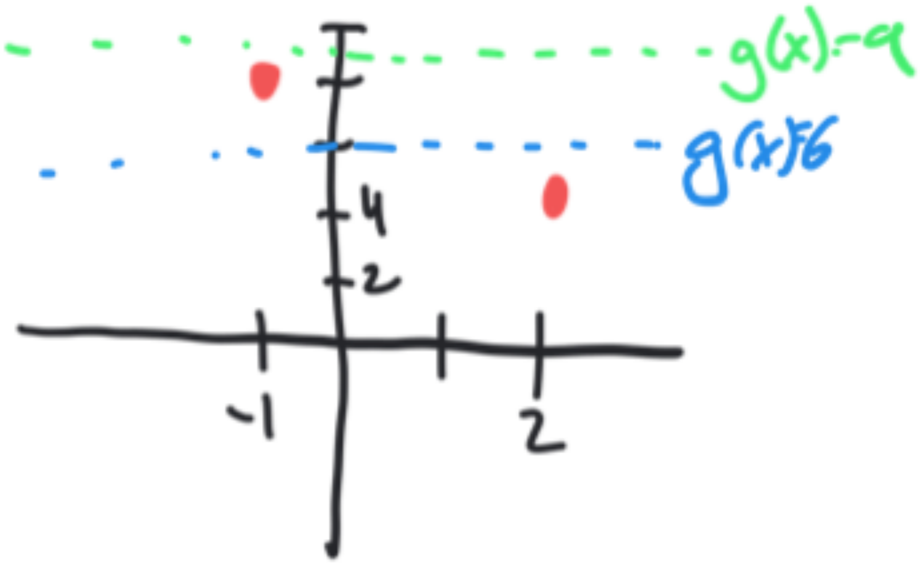
$\boxed{g(x) = 9 - x^2}$, $[-1, 2]$, $g(c) = 9$

g(x) is continuous on $[-1, 2]$

$g(-1) = 9 - (-1)^2 = 8$

$g(2) = 9 - (2)^2 = 5$

$\boxed{\text{IVT does not apply}}$



what if $[-1, 2]$ $g(c) = 6$

IVT does Apply

$\boxed{c = \sqrt{3}}$

$9 - x^2 = 6$

$-9 \qquad -9$

$\frac{-x^2}{-1} = \frac{-3}{-1}$

$\sqrt{x^2} = \sqrt{3}$

$x = \pm\sqrt{3}$

$\overset{1}{\sqrt{1}}, \sqrt{3}, \overset{2}{\sqrt{4}}$

$\sqrt{3} \approx 1.7$