

① Calculate the derivative with the limit process:  $f(x) = 2 - 5x$

$$\lim_{\Delta x \rightarrow 0} \frac{(2 - 5(x + \Delta x)) - (2 - 5x)}{\Delta x}$$

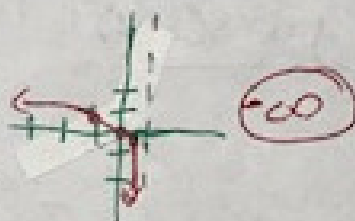
$$\lim_{\Delta x \rightarrow 0} \frac{2 - 5x - 5\Delta x - 2 + 5x}{\Delta x}$$

$$\lim_{\Delta x \rightarrow 0} \frac{-5\Delta x}{\Delta x} = \lim_{\Delta x \rightarrow 0} -5 = \boxed{-5}$$

②  $\lim_{x \rightarrow 1^-} \frac{x}{x-1}$

x	-1	0	1/2	1
y	1/2	0	-1	?

$$\frac{1/2}{1/2 - 1} = \frac{1/2}{-1/2} = -1$$



③  $g(x) = \sqrt{7-x}$  On what interval(s) is  $f(g(x))$  continuous?  
 $f(x) = \frac{x-9}{x^2+2}$

$$f(g(x)) = \frac{\sqrt{7-x}}{(\sqrt{7-x})^2 + 2} = \frac{\sqrt{7-x}}{7-x+2} = \frac{\sqrt{7-x}}{-x+9}$$

$$\begin{array}{l} 7-x \geq 0 \\ \rightarrow -x \geq -7 \\ \rightarrow x \leq 7 \end{array} \quad \& \quad \begin{array}{l} -x+9 \neq 0 \\ \rightarrow -x \neq -9 \\ \rightarrow x \neq 9 \end{array}$$

$$\boxed{x \leq 7} \quad \& \quad \boxed{x \neq 9}$$

④ Find  $\frac{d^2 y}{dx^2}$  for  $y = \frac{x}{x-1}$

$$\frac{dy}{dx} = \frac{1(x-1) - (x)(1)}{(x-1)^2} = \frac{x-1-x}{(x-1)^2} = \frac{-1}{(x-1)^2} = -1(x-1)^{-2}$$

$$\frac{d^2 y}{dx^2} = 2(x-1)^{-3} (1) = \boxed{\frac{2}{(x-1)^3}}$$

⑤  $f(x) = (5x-1)^3$  find  $f''(x)$

$$f'(x) = 3(5x-1)^2(5) = 15(5x-1)^2$$

$$f''(x) = 30(5x-1)'(5) = \boxed{150(5x-1)}$$

⑥  $f(3) = 1$   $f'(3) = 5$

Find the equation of the tangent line at  $x=3$  of  $f(x)$ .

$$y-1 = 5(x-3)$$

$$y-1 = 5x-15$$

$$\boxed{y = 5x-14 \text{ or } y-1 = 5(x-3)}$$

⑦  $f(x) = \frac{x+1}{x^2-1}$  Find all critical numbers.

$$f(x) = \frac{x+1}{(x-1)(x+1)} = \frac{1}{x-1} = (x-1)^{-1} \text{ hole at } x=-1$$

$$f'(x) = -(x-1)^{-2} = \frac{-1}{(x-1)^2}$$

$$\sqrt{(x-1)^2} = |x-1|$$

$x-1 = \pm 0$

$x=1$  is a VA

No critical #s

⑧  $g(x) = 2x^4 - x^2$  a) At what  $x$  values does  $g(x)$  have a Rel. Min?

b) Find  $x$  values of all Pot I of  $g(x)$ .

$$g'(x) = 8x^3 - 2x = 2x(4x^2 - 1)$$

$$g''(x) = 24x^2 - 2 = 2(12x^2 - 1)$$

$$12x^2 - 1 = 0$$

$$\sqrt{x^2} = \sqrt{\frac{1}{12}}$$

$$x = \pm \frac{1}{\sqrt{12}}, \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{12}}{12} = \frac{2\sqrt{3}}{12} = \boxed{\pm \frac{\sqrt{3}}{6}}$$

$$2x=0 \quad 4x^2-1=0$$

$$\boxed{x=0} \quad \sqrt{x^2} = \sqrt{\frac{1}{4}} \quad \boxed{x = \pm \frac{1}{2}}$$

$$g''(0) = - \wedge g''(\frac{1}{2}) = + \wedge g''(-\frac{1}{2}) = +$$

$x = \frac{1}{2}$  &  $-\frac{1}{2}$  are both Rel. Min

$x < \frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{6} < x < \frac{\sqrt{3}}{6}$	$x > \frac{\sqrt{3}}{6}$
+	-	+

$x = \pm \frac{\sqrt{3}}{6}$  are Pot I

9) You measure the radius of a circle as 7 inches and you are within  $\pm 2$  inches. Find the propagated error of the area of the circle.

$$r = 7 \quad dr = \pm 2$$


$$A = \pi r^2$$

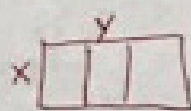
$$dA = 2\pi r dr$$

$$dA = 2\pi(7)(\pm 2) = \pm 28\pi$$

$$\frac{dA}{A} = \frac{28\pi}{49\pi} = \frac{4}{7}$$

$$dA = \pm 2.8\pi$$

10)  What is the maximum area possible with 200 ft of fencing



$$4x + 2y = 200$$

$$2y = -4x + 200$$

$$y = -2x + 100$$

$$y = -2x + 100$$

$$A = xy$$

$$A = x(-2x + 100)$$

$$A = -2x^2 + 100x$$

$$A'(x) = 4x + 100 \quad A''(x) = 4 = - \text{Max}$$

$$0 = 4x + 100$$

$$x = 25$$

$$A(25) = 25(-2(25) + 100)$$

$$A(25) = 25(50) = 1250 \text{ ft}^2$$

11) Find the horizontal asymptote for each

a)  $y = \frac{5x-2}{x^2-3}$

b)  $f(x) = \frac{7x-5}{x+3}$

c)  $g(x) = \frac{3x}{\sqrt{5+x}}$

$$\lim_{x \rightarrow \infty} \frac{\frac{5}{x} - \frac{2}{x^2}}{1 - \frac{3}{x^2}} = 0$$

$$\lim_{x \rightarrow \infty} \frac{7 - \frac{5}{x}}{1 + \frac{3}{x}} = 7$$

$$\lim_{x \rightarrow \infty} \frac{3\sqrt{x}}{\sqrt{\frac{5}{x} + 1}} = \infty$$

a)  $y = 0$

b)  $y = 7$

c) None