

## 2<sup>nd</sup> Semester Algebra 2 Notes

### 9-7 Probability of Multiple Events

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Dependent Events: First event affects the outcome of a second event.

Independent Events: First event does *not* affect the outcome of a second event.

Mutually Exclusive Events: Two events *cannot* happen at the same time.

$$P(A \text{ or } B) = P(A) + P(B)$$

Not Mutually Exclusive Events: Two events *can* happen at the same time.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$P(A) = \frac{2}{5}$ ,  $P(B) = 80\%$ ,  $P(C) = 0.20$  (A, B & C are independent events)

- 1) P(A and C)                      2) P(A or C) if Mutually Exclusive      3) P(A or B) if *not* Mutually Exclusive

3 Large Blue Marbles, 2 Large Green Marbles, 2 Small Blue, and 5 Small Green Marbles

4) P(Large or Green)

5) With Replacement: P(a Blue then a Small)

6) Without Replacement: P(Two Blue)

7) Without Replacement: P(a Large then a Small)

### 12-2 Conditional Probability

$P(B | A)$  = Probability of event B, given event A

	Male	Female
Freshmen		
Sophomore		
Juniors		
Seniors		

1) P(Male)

2) P(Junior | Female)

3) P(Male | Sophomore)

4) P(Sophomore | Freshman)

Survey: 20% of the people were left-handed, 30% of the left-handed people are males, and 60% of the right-handed people are females.

5) P(male | right-handed)

6) P(left-handed female)

**12-4 Standard Deviation:** How much values in a data set vary (deviate) from the mean.

$$\text{Standard Deviation} = \sigma = \sqrt{\frac{(1^{\text{st}} \# - \text{Mean})^2 + (2^{\text{nd}} \# - \text{Mean})^2 + \dots + (\text{Last} \# - \text{Mean})^2}{n}}, n = \# \text{ of terms}$$

1. Standard Deviation: 7, 12, 1, 4

2. Which of the following set of data will probably have the *largest* standard deviation and which will have the *smallest*?

- a) 13, 20, 10, 5, 27, 12      b) 26, 21, 24, 23, 25, 17      c) 10, 15, 30, 41, 2, 1      d) 18, 90, 87, 31

3. Variance: 10, 2, 7, 3, 13 (Variance =  $\sigma^2$ )

**Exponent Properties**

$$a^m \cdot a^n = \quad x^3 \cdot x^2 = \quad (-5x^3y)(6x^2y^7) =$$

$$(a^m)^n = \quad (x^2)^3 = \quad 3x^7(x^5)^2 =$$

$$\frac{a^m}{a^n} = \quad \frac{x^5}{x^3} = \quad \frac{6x^6y^8}{9x^3y^7} =$$

$$a^{-n} = \quad \frac{x^2}{x^5} = \quad 3^{-2} = \quad 5x^{-4}y =$$

$$(ab)^n = \quad (-2x)^3 = \quad (x^5y^{-3})^{-4} =$$

$$\left(\frac{a}{b}\right)^n = \quad \left(\frac{x}{5}\right)^2 = \quad \left(\frac{x^3}{y}\right)^{-5} =$$

$$a^0 = \quad \frac{x^2}{x^2} = \quad 3x^0 = \quad (-5x)^0 =$$

**Radical Expressions and Rational Exponents**

$$x^{a/b} = \sqrt[b]{x^a} = \left(\sqrt[b]{x}\right)^a \quad \text{Radical Notation:}$$

Rational Exponent Notation:

1. Simplify and rewrite in radical notation:  $x^{4/6}$

2. Simplify and rewrite in rational exponent notation:  $\sqrt[4]{x^{10}}$

3. Simplify:  $36^{-3/2}$

**7-5 Solving Square Roots**

Solve:  $\sqrt{2x-3} + 7 = 11$

Chapter 8: Exponential and Logarithmic Functions

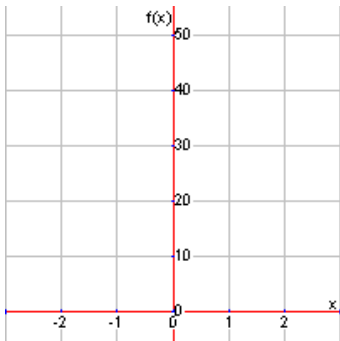
8-1 Exploring Exponential Models

Exponential Function:  $y = ab^x$ ,  $a =$

$b =$

Exponential Growth:

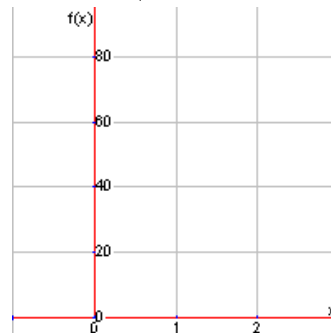
1)  $f(x) = 3(4)^x$



$x$	$y$	Calculations

Exponential Decay:

2)  $y = 80(\frac{1}{4})^x$



$x$	$y$	Calculations

Determine if they are Exponential Growth or Decay:

3)  $y = 8(3.5)^x$

4)  $y = 0.9^x$

5)  $y = \frac{1}{2}(\frac{5}{4})^x$

6) You own a \$10,000 car and it is decreasing in value at about 15% each year.  $A = P(1 - r)^t$   $A=Final Amount$

$P=Principal(Initial Amount)$   $r=rate$   $t=time(yrs)$

- a) Decay Factor      b) Function      c) How much is it worth in 8 years?

7) A soda presently costs \$0.50. Inflation increases the cost of things by about 3% each year.  $A = P(1 + r)^t$

- a) Growth Factor      b) Function      c) How much does it cost in 20 years?

8-3 Logarithmic & Exponential Transformations

1)  $\log M = \log_{10} M$

2)  $\log_b b = 1$

3)  $b^{\log_b x} = x$

1)  $9^2 = 81$

2)  $\frac{1}{25} = 5^{-2}$

3)  $\log_4 64 = 3$

4)  $\log 10,000 = 4$

5)  $\log_4 16$

6)  $\log_5 (\frac{1}{25})$

7)  $\log 100$

### 8-4 Properties of Logarithms

$$1) \log_b(MN) = \log_b M + \log_b N \quad 2) \log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N \quad 3) \log_b M^N = N \cdot \log_b M$$

1) Condense:

a)  $\log 7 + 3\log x - 4\log y$

b)  $\frac{1}{2}(\log 25 + \log 36) - 8\log x$

2) Expand:

a)  $\log_4\left(\frac{4}{x^2y}\right)$

b)  $\log_2 3\sqrt{8a^5}$

3) Evaluate:  $2\log_2 8 - \log_2 4$

4) Given:  $\log 5 \approx 0.7$  and  $\log 6 \approx 0.8$   
Find:  $\log 150$

### 8-5 Exponential and Logarithmic Equations

$$\text{Change of Base Formula: } \log_b M = \frac{\log M}{\log b}$$

1)  $2(7)^{2x-5} - 6 = 24$

2)  $5^{8x-5} = 25^{3x+1}$

3)  $2\log_5(3x-2)+1=7$

4)  $\log(2x-7)-\log 3=1$

**8-6 Natural Logarithms**  $e \approx$ 

1)  $\ln M = \log_e M$

2)  $\log_e e = 1$

3)  $e^{\log_e x} = x$

1) Condense:  $2\ln 5 - \frac{1}{2}\ln x + \ln y$

2) Simplify:  $3 - 4\ln e^5$

3)  $\ln(3n+2)^2 - 4 = 10$

4)  $e^{5x+1} - 2 = 7$

5)  $\ln(x+5) + \ln(x-5) = 0$

**Compounding Continuously:**  $A = Pe^{rt}$ 

6) You put \$200 in a CD and it will be worth \$300 in 4 years. What type of interest rate are you getting if interest is compounded continuously?

**Chapter 9 Rational Functions****Fractions**

1)  $\frac{3}{4} \cdot \frac{10}{9}$

2)  $\frac{2}{3} \div \frac{12}{5}$

3)  $\frac{3}{4} - \frac{5}{6}$

4)  $\frac{3}{4} + \frac{2}{3}$

**Factoring**

1) $-12x^3 + 8x^2$	2) $2x^2 + 10x + 12$	3) $x^2 + 2xy - 8y^2$	4) $6x^2 - 5x - 4$
5) $4x^6 - 81y^2$	6) $100x^2 - 140x + 49$	7) $x^3 + 27$	8) $8x^6 - 125y^3$

**9-4 Rational Expressions**

1) $\frac{12x^5 y^{-6}}{4x^{-4} y^2}$	2) $\frac{x^2 - 16}{5x - 20}$	3) $\frac{30x^6}{5y^{-4}} \div \frac{3x^{-2}}{10y}$
4) $\frac{2x^2 - 2}{4x} \div \frac{x^2 + 3x + 2}{2x^2 + 4x}$	5) $\frac{x - 1}{x^2 - 5x + 4} \cdot (12 - 3x)$	6) $\frac{\frac{3x^5}{x - 5}}{\frac{6x^3}{2x - 10}}$

**9-5 Adding and Subtracting Rational Expressions**

1) $\frac{x^2}{4x} + \frac{3x}{4x}$	2) $\frac{x^2 + 2}{x^2 - 9} - \frac{11}{x - 3}$	3) $\frac{x}{x^2 - x - 2} + \frac{x + 5}{x^2 - 1}$
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### 9-6 Solving Rational Equations

1) $\frac{x-3}{x+4} = \frac{2}{x}$	2) $\frac{x}{4} - x = \frac{3}{2x}$	3) $\frac{3}{x^2-4} - \frac{4}{x+2} = \frac{1}{x-2}$
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## Chapter 11 – Sequences and Series

### 11-2 Arithmetic Sequences

$$a_n = a_0 + dn, \quad d = \text{common difference}, \quad n = \text{which term}, \quad a_0 = \text{term before the 1}^{st} \text{ term}$$

Is the sequence arithmetic? If so, identify the common difference.

1) 3, 10, 17, ...

2) 6, -2, -10, ...

3) 1, -2, 5, ...

4) Write an equation for the sequence:  
-2, 5, 12, ...

5) Find the 32<sup>nd</sup> term of the sequence:  
8, 11, 14, ...

6) Find the 17<sup>th</sup> term of the sequence:  
 $a_{18} = 18, d = 5$

### 11-3 Geometric Sequences

$$a_n = a_0(r)^n, \quad r = \text{common ratio}, \quad n = \text{which term}, \quad a_0 = \text{term before the 1}^{st} \text{ term}$$

What type of sequence? If it is geometric, then what is the common ratio and the next 2 terms.

1) 3, 6, 12, ...

2) -6, -2, 2, ...

3) 90, -30, 10, ...

4) Write an equation for the sequence. Generate the first 5 terms: $a_1 = 2, r = -3$	5) Find the 10 <sup>th</sup> term of the sequence: $a_9 = 180, r = \frac{1}{3}$	6) Find the 10 <sup>th</sup> term of the sequence: $a_{12} = -36, r = 2$
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#### 11-4 Arithmetic Series

$$S_n = \frac{n}{2}(a_1 + a_n), \quad n = \text{which term}, \quad a_1 = 1^{\text{st}} \text{ term}, \quad a_n = \text{last term}$$

Tell if it is a *sequence* or *series*. Tell if it is *infinite* or *finite*.

1)  $3 + 8 + 13 + 18$

2)  $7, 5, 3, \dots$

3)  $13 + 12 + 11 + \dots$

4) Write the related series. Evaluate the series: $-12, -2, 8, 18$	5) The sequence has 8 terms. Evaluate the series: $2, 6, 10, \dots, 30$	6) Evaluate the series to the 7 <sup>th</sup> term: $21 + 18 + 15 + \dots$
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#### 11-5 Geometric Series

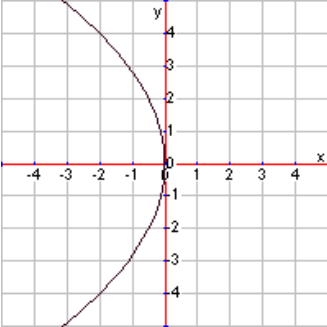
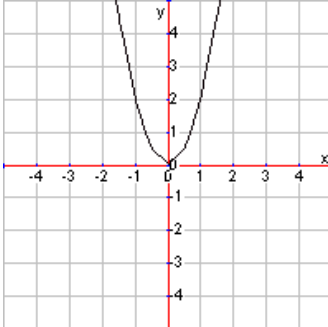
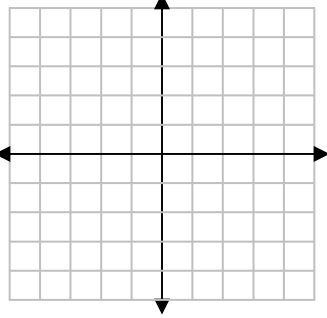
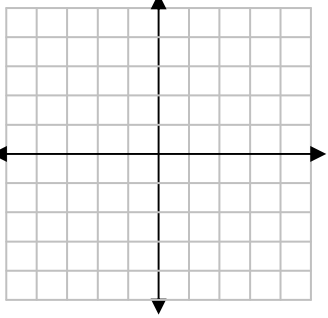
$$\text{Finite Series: } S_n = \frac{a_1(1-r^n)}{1-r}$$

$$\text{Infinite Series: } S_\infty = \frac{a_1}{1-r}$$

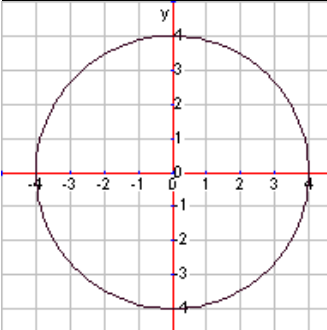
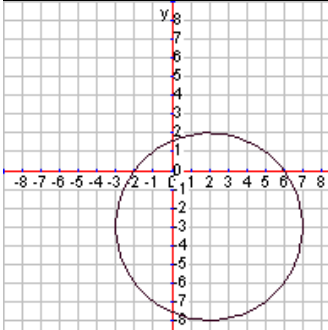
1) Evaluate for the specified number of terms: $1 + 3 + 9 + \dots ; n = 7$	Which series have a sum? 2) $120 + 60 + 30 + \dots$  3) $5 + 15 + 45 + \dots$
4) Evaluate the infinite geometric series: $3 + \frac{1}{3} + \frac{1}{27} + \dots$	5) Is the series <i>arithmetic</i> or <i>geometric</i> ? Evaluate for the specified number of terms. $-3 + 12 - 48 + \dots ; n = 6$

Chapter 10 – Quadratic Relations and Conic Sections

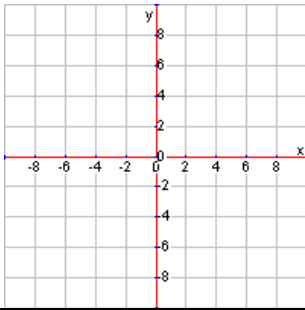
10-2 Parabolas

<u>Opens Horizontally</u>			<u>Opens Vertically</u>		
<u>Equation</u>	<u>Focus: <math>c = \frac{1}{4a}</math></u>	<u>Directrix</u>	<u>Equation</u>	<u>Focus: <math>c = \frac{1}{4a}</math></u>	<u>Directrix</u>
$x = ay^2$	$(c, 0)$	$x = -c$	$y = ax^2$	$(0, c)$	$y = -c$
$x = -\frac{1}{8}y^2$			$y = 2x^2$		
					
<p>1) Identify the focus and directrix. <math>y = 3x^2</math></p>			<p>2) Identify the focus and directrix. <math>x = -\frac{1}{12}y^2</math></p>		
<p>3) Sketch the graph and identify the vertex, focus, and directrix: <math>x = y^2</math></p> <div style="text-align: center;">  </div>			<p>4) Sketch the graph and identify the vertex, focus, and directrix: <math>x^2 = -4y</math></p> <div style="text-align: center;">  </div>		

10-3 Circles

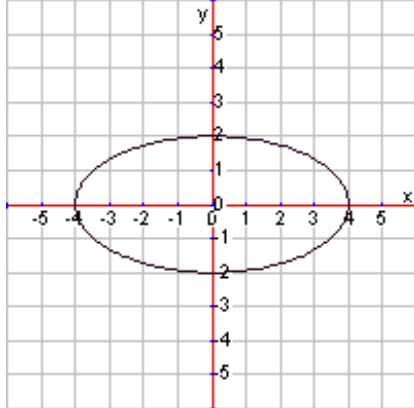
<u>Center is at origin</u>			<u>Center is at (h,k)</u>		
<u>Equation</u>	<u>Center</u>	<u>Radius</u>	<u>Equation</u>	<u>Center</u>	<u>Radius</u>
$x^2 + y^2 = r^2$	$(h, k)$	$r = \sqrt{r^2}$	$(x-h)^2 + (y-k)^2 = r^2$	$(h, k)$	$r = \sqrt{r^2}$
$x^2 + y^2 = 16$			$(x-2)^2 + (y+3)^2 = 25$		
					

1) Graph:  $(x+3)^2 + (y-1)^2 = 9$



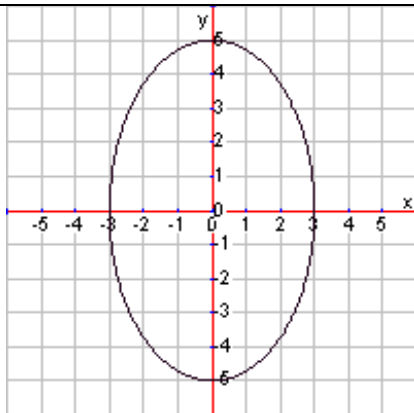
2) Find the center and radius:  $(x-7)^2 + (y+8)^2 = 60$

**10-4 Ellipses**



Horizontal Major Axis

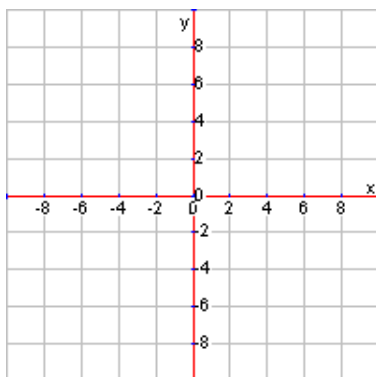
<u>Equation</u> <i>a</i> <sup>2</sup> is largest denominator	<u>Vertices</u> <i>on major axis</i>	<u>Co-Vertices</u> <i>on minor axis</i>	<u>Foci</u> $c^2 = a^2 - b^2$ <i>on major axis</i>
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$(\pm a, 0)$	$(0, \pm b)$	$(\pm c, 0)$
$\frac{x^2}{16} + \frac{y^2}{4} = 1$			



Vertical Major Axis

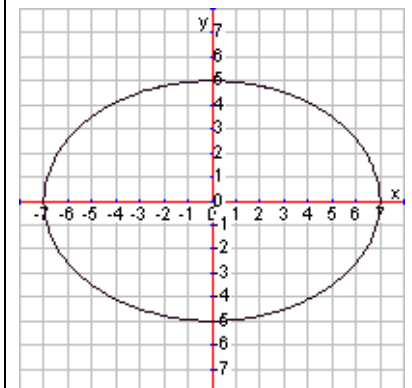
<u>Equation</u> <i>a</i> <sup>2</sup> is largest denominator	<u>Vertices</u> <i>on major axis</i>	<u>Co-Vertices</u> <i>on minor axis</i>	<u>Foci</u> $c^2 = a^2 - b^2$ <i>on major axis</i>
$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	$(0, \pm a)$	$(\pm b, 0)$	$(0, \pm c)$
$\frac{x^2}{9} + \frac{y^2}{25} = 1$			

1) Graph and label the foci:  $\frac{x^2}{25} + \frac{y^2}{16} = 1$

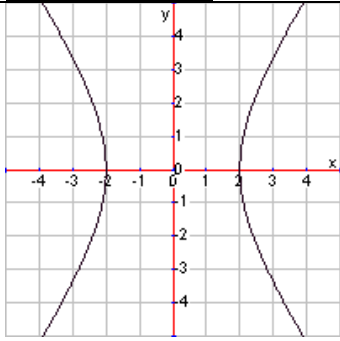


2) Find the foci:  $11x^2 + 2y^2 = 22$

3) Write the equation for the graph:

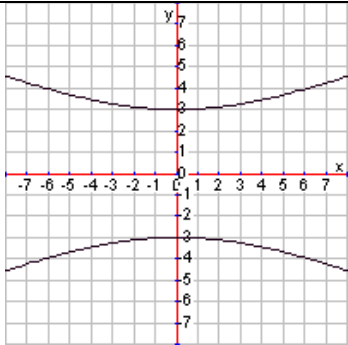


### 10-5 Hyperbolas



#### Opens Horizontally

Equation <i>a</i> <sup>2</sup> is the denominator	Vertices	Asymptotes	Foci <i>c</i> <sup>2</sup> = <i>a</i> <sup>2</sup> + <i>b</i> <sup>2</sup> follow vertices
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	(± <i>a</i> , 0)	$y = \pm \frac{b}{a}x$	(± <i>c</i> , 0)
$\frac{x^2}{4} - \frac{y^2}{9} = 1$			

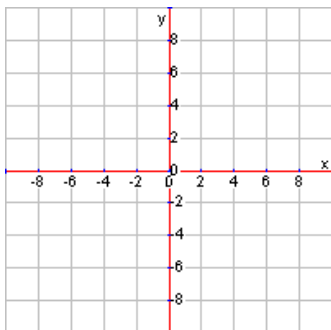


#### Opens Vertically

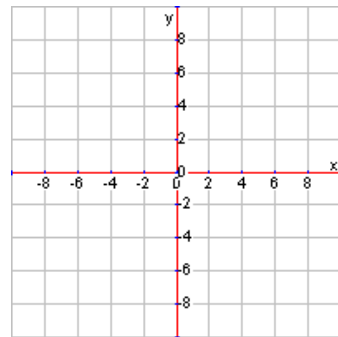
Equation <i>a</i> <sup>2</sup> is the denominator	Vertices	Asymptotes	Foci <i>c</i> <sup>2</sup> = <i>a</i> <sup>2</sup> + <i>b</i> <sup>2</sup> follow vertices
$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$	(0, ± <i>a</i> )	$y = \pm \frac{a}{b}x$	(0, ± <i>c</i> )
$\frac{y^2}{9} - \frac{x^2}{49} = 1$			

1) Graph and label the foci:

$$\frac{y^2}{9} - \frac{x^2}{16} = 1$$



2) Graph:  $9x^2 - y^2 = 9$

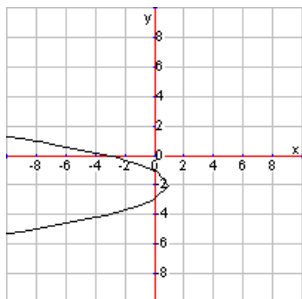


3) Find the vertices and asymptotes:

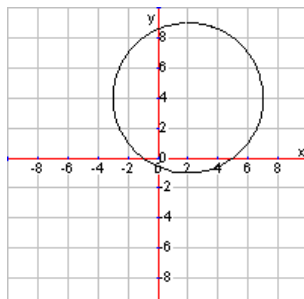
$$9y^2 - 25x^2 = 225$$

### 10-6 Translating Conic Sections

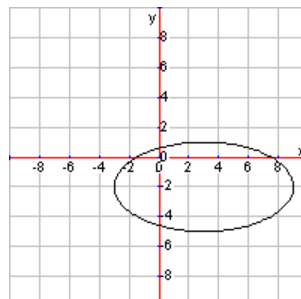
1) Write the equation:



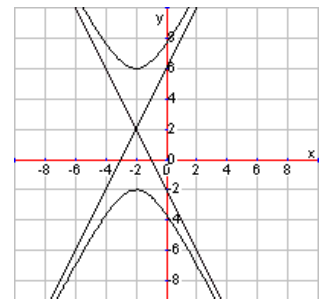
2) Write the equation:



3) Write the equation:



4) Write the equation:



5) Write the equation for an ellipse with a center at  $(-2,7)$  and a vertical major axis of 16 and a horizontal minor axis of 6.

Classifying Conic Sections

$$ax^2 + by^2 + cx + dy + e = 0$$

Parabola:

Circle:

Ellipse:

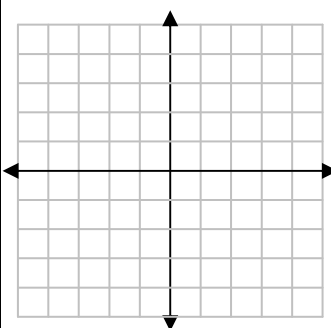
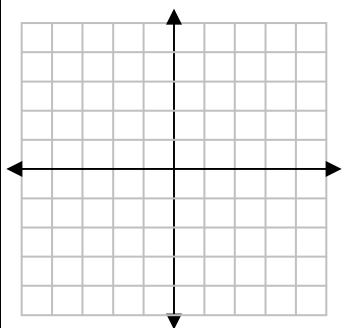
Hyperbola:

6) Classify the conic. Change it into standard form and then graph it. Find the center/vertex. Find and label foci and/or other important points on the graph.

$$y^2 - x - 4y + 1 = 0$$

7) Classify the conic. Change it into standard form and then graph it. Find the center/vertex. Find and label foci and/or other important points on the graph.

$$x^2 + y^2 - 2x + 6y = -6$$



8) Classify the conic. Change it into standard form and then graph it. Find the center/vertex. Find and label foci and/or other important points on the graph.

$$x^2 + 9y^2 - 4x + 18y = -4$$

9) Classify the conic. Change it into standard form and then graph it. Find the center/vertex. Find and label foci and/or other important points on the graph.

$$-4x^2 + 9y^2 + 24x - 72 = 0$$

