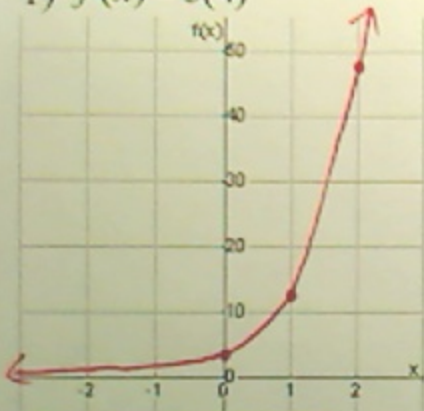


8-1 Exploring Exponential Models

Exponential Function: $y = ab^x$, $a =$ *Initial Amount* $b =$ *Growth/Decay Factor*

Exponential Growth: $b > 1$

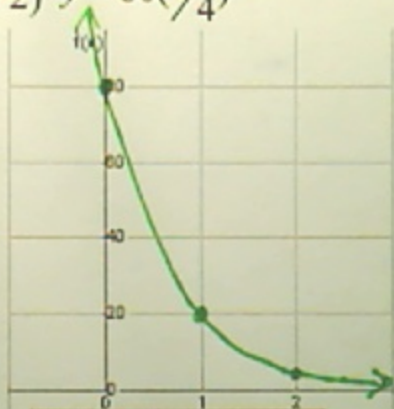
1) $f(x) = 3(4)^x$



x	y	Calculations
0	3	$3(4)^0 = 3 \cdot 1$
1	12	$3(4)^1 = 3 \cdot 4$
2	48	$3(4)^2 = 3(16)$

Exponential Decay: $b < 1$ & $b > 0$

2) $y = 80\left(\frac{1}{4}\right)^x$



x	y	Calculations
0	80	$80\left(\frac{1}{4}\right)^0 = 80(1)$
1	20	$80\left(\frac{1}{4}\right)^1 = 80\left(\frac{1}{4}\right)$
2	5	$80\left(\frac{1}{4}\right)^2 = 80\left(\frac{1}{16}\right)$

Determine if they are Exponential Growth or Decay:

3) $y = 8(\underline{3.5})^x$

Growth

8 Initial Value

4) $y = 0.9^x$

$y = 1(\underline{.9})^x$

Decay

1 Initial Value

5) $y = \frac{1}{2}(\underline{\frac{5}{4}})^x$

 $1\frac{1}{4}$ Growth

6) You own a \$10,000 car and it is decreasing in value at about 15% each year. $A = P(1-r)^t$ $A=Final\ Amount$ $P=Principal(Initial\ Amount)$ $r=rate$ $t=time(yrs)$

a) Decay Factor b) Function c) How is it worth in 8 years?

$$1 - .15$$

$$(0.85)$$

$$A = 10,000(1 - .15)^t$$

$$A = 10,000(.85)^t$$

$$A = 10,000(.85)^8$$

$$(\$2,724.91)$$

7) A soda presently costs \$0.50. Inflation increases the cost of things by about 3% each year. $A = P(1+r)^t$

a) Growth Factor b) Function c) How much cost in 20 years?

$$1 + .03$$

$$(1.03)$$

$$A = 0.50(1.03)^t$$

$$A = 0.5(1.03)^{20}$$

$$(\$.90)$$

8-3 Logarithmic & Exponential Transformations

1) $\log M = \log_{10} M$

2) $\log_b b = 1$

3) $b^{\log_b x} = x$

1) $9^2 = 81$

2) $\frac{1}{25} = 5^{-2}$

3) $\log_4 64 = 3$

4) $\log_{10} 10,000 = 4$

$\log_9 81 = 2$

$5^{-2} = \frac{1}{25}$

$4^3 = 64$

$10^4 = 10,000$

$\log_5 \frac{1}{25} = -2$

5) $\log_4 16 = x$

6) $\log_5 \left(\frac{1}{25}\right) = x$

7) $\log_{10} 100 = x$

$4^x = 16$

$5^x = \frac{1}{25}$

$10^x = 100$

$x = 2$

$x = -2$

$x = 2$

8-4 Properties of Logarithms

$$1) \log_b(MN) = \log_b M + \log_b N \quad 2) \log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N \quad 3) \log_b M^N = N \cdot \log_b M$$

1) Condense:

a) $\log 7 + 3 \log x - 4 \log y$

$$\log 7 + \log x^3 - \log y^4$$

$$\log(7x^3) - \log y^4$$

$$\log\left(\frac{7x^3}{y^4}\right)$$

b) $\frac{1}{2}(\log 25 + \log 36) - 8 \log x$

$$\log 25^{\frac{1}{2}} + \log 36^{\frac{1}{2}} - \log x^8$$

$$\log 5 + \log 6 - \log x^8$$

$$\log 30 - \log x^8$$

$$\log\left(\frac{30}{x^8}\right)$$

2) Expand:

a) $\log_4 \left(\frac{4}{x^2 y} \right)$

$$\log_4 4 - \log_4 x^2 - \log_4 y$$

$$1 - 2\log_4 x - \log_4 y$$

b) $\log_2 3\sqrt{8a^5} \quad \log_2 (3(8a^5)^{1/2})$

$$\log_2 3 + \log_2 (8a^5)^{1/2}$$

$$\log_2 3 + \frac{1}{2} \log_2 (8a^5)$$

$$\log_2 3 + \frac{1}{2} (\log_2 8 + \log_2 a^5)$$

$$\log_2 3 + \frac{1}{2} \log_2 8 + \frac{1}{2} \log_2 a^5$$

$$\log_2 3 + \frac{3}{2} + \frac{5}{2} \log_2 a$$

3) Evaluate: $2^{\log_2 8} - \log_2 4$

$$\log_2 8 = x$$

$$2^x = 8$$

$$x = 3$$

$$2^x = 4$$

$$x = 2$$

$$2(3) - 2 = \textcircled{4}$$

$$\log_2 8^2 - \log_2 4$$

$$\log_2 \left(\frac{64}{4} \right)$$

$$\log_2 16 = x$$

$$2^x = 16$$

$$\textcircled{4}$$

4) Given:

$$\log_5 \approx 0.7 \text{ and } \log_6 \approx 0.8$$

Find: $\log 150$

$$\log(5 \cdot 5 \cdot 6)$$

$$\log 5 + \log 5 + \log 6$$

$$.7 + .7 + .8$$

$$1.4 + .8 = \textcircled{2.2}$$

8-5 Exponential and Logarithmic Equations

Change of Base Formula: $\log_b M = \frac{\log M}{\log b}$

$$1) 2(7)^{2x-5} - 6 = 24$$

$$\begin{array}{r} 2(7)^{2x-5} = 30 \\ \hline 2 \end{array}$$

$$7^{2x-5} = 15$$

$$\log_7 15 = 2x - 5$$

$$\frac{\log 15}{\log 7} \approx 1.4$$

$$\begin{array}{r} 1.4 = 2x - 5 \\ +5 \quad +5 \end{array}$$

$$\frac{6.4}{2} = \frac{2x}{2} \quad \boxed{x = 3.2}$$

$$2) 5^{8x-5} = 25^{3x+1}$$
$$(5^2)^{3x+1} \quad 2(3x+1)$$

$$5^{8x-5} = 5^{6x+2}$$

$$\begin{array}{r} 8x - 5 = 6x + 2 \\ -6x \quad -6x \end{array}$$

$$\begin{array}{r} 2x - 5 = 2 \\ +5 \quad +5 \end{array}$$

$$\begin{array}{r} 2x = 7 \\ \hline 2 \end{array} \quad \boxed{x = \frac{7}{2}}$$

$$3) 2 \log_5(3x-2) + 1 = 7$$

$$\frac{2 \log_5(3x-2)}{2} = \frac{6}{2}$$

$$\log_5(3x-2) = 3$$

$$5^3 = 3x-2$$

$$125 = 3x-2$$

$$+2 \quad +2$$

$$\frac{127}{3} = \frac{3x}{3}$$

$$x = \frac{127}{3}$$

$$4) \log(2x-7) - \log 3 = 1$$

$$\log\left(\frac{2x-7}{3}\right) = 1$$

$$10^1 = \frac{2x-7}{3}$$

$$3 \cdot 10 = \frac{2x-7}{3} \cdot 3$$

$$30 = 2x-7$$

$$+7 \quad +7$$

$$\frac{37}{2} = \frac{2x}{2}$$

$$x = \frac{37}{2}$$

8-6 Natural Logarithms $e \approx 2.718$

$$1) \ln M = \log_e M$$

$$2) \log_e e = 1$$

$$3) e^{\log_e x} = x$$

1) Condense: $2 \ln 5 - \frac{1}{2} \ln x + \ln y$ 2) Simplify: $3 - 4 \ln e^5$

$$\ln 5^2 - \ln x^{1/2} + \ln y$$

$$\ln 25 - \ln \sqrt{x} + \ln y$$

$$\ln \left(\frac{25y}{\sqrt{x}} \right)$$

$$3 - 4(5 \ln e) \quad \log_e e = 1$$

$$3 - 4(5 \cdot 1)$$

$$3 - 4(5)$$

$$3 - 20 = (-17)$$

$$3) \ln(3n+2)^2 - 4 = 10$$

$$2 \ln(3n+2) - 4 = 10$$

+4 +4

$$\frac{2 \ln(3n+2)}{2} = \frac{14}{2}$$

$$\ln_e(3n+2) = 7$$

$$e^7 = 3n+2$$

$$\frac{e^7 - 2}{3} = n$$

$$n = \frac{e^7 - 2}{3}$$

$$4) e^{5x+1} - 2 = 7$$

$$+2 \quad +2$$

$$e^{5x+1} = 9$$

$$\log_e 9 = 5x+1$$

$$\ln 9 = 5x+1$$

$$\frac{\ln 9 - 1}{5} = x$$

$$x = \frac{-1 + \ln 9}{5}$$

$$5) \ln(n+5) + \ln(n-5) = 0$$

$$\ln((n+5)(n-5)) = 0$$

$$e^0 = (n+5)(n-5)$$

$$1 = n^2 - 25$$

+25 +25

$$\sqrt{n^2} = \sqrt{26}$$

$$n = \pm \sqrt{26}$$

± 5.1

Compounding Continuously: $A = Pe^{rt}$

6) You put \$200 in a CD and it will be worth \$300 in 4 years. What type of interest rate are you getting if interest is compounded continuously?

$$A=300 \quad P=200 \quad r=? \quad t=4$$

$$\frac{300}{200} = \frac{200e^{r \cdot 4}}{200}$$

$$\frac{3}{2} = e^{4r}$$

$$e^{4r} = \frac{3}{2}$$

$$\log_e(1.5) = 4r$$

$$\frac{\ln(1.5)}{4} = \frac{4r}{4}$$

$$r = \frac{\ln(1.5)}{4}$$

$$r \approx .101$$

$$10\%$$