11-2 Arithmetic Sequences

$a_n = a_0 + dn$, d = common difference, n = which term, a_0 = term before the 1st term

Is the sequence arithmetic? If so, identify the common difference.

1) 3, 10, 17,…

$d = 10 - 3 = 7$

$d = 7$ yes

2) 6, -2, -10, …

$d = -2 - 6 = -8$

yes

3) 1, -2, 5, …

(-3, 7)

$d = -2 - 1 = -3$

Not

4) Write an equation for the sequence: -2, 5, 12, …

$a_0 = -9$

$a_n = -9 + 7n$

$d = 5 - -2 = 7$

$d = 7$

5) Find the 32nd term of the sequence: 8, 11, 14, …

$a_n = a_0 + dn$ $\quad d = 11 - 8 = 3$

$a_n = 5 + 3n$ $\quad a_0 = 5$

$a_{32} = 5 + 3(32)$

$a_{32} = 5 + 96 = 101$

6) Find the 17th term of the sequence: $a_{18} = 18,\ d = 5$

13, 18, 23

$a_{17} = 13$

11-3 Geometric Sequences

$a_n = a_0 r^n$, r = common ratio, n = which term, a_0 = term before the 1st term

What type of sequence? If it is geometric, then what is the common ratio and the next 2 terms.

1) 3, 6, 12, … Geo

$r = \frac{6}{3} = 2$

$\frac{24}{4th}, \frac{48}{5th}$

2) -6, -2, 2, … Arith

$d = -2 - -6 = 4$

$\frac{6}{4th}, \frac{10}{5th}$

3) 90, -30, 10, … Geo

$r = \frac{-30}{90} = -\frac{1}{3}$

$-\frac{10}{3}, \frac{10}{-9}, \frac{-10}{3}, \frac{-1}{3}$

4) Write an equation for the sequence. Generate the first 5 terms:

$a_1 = 2, r = -3$ rule

$a_0 = -\frac{2}{3}$

$a_n = -\frac{2}{3}(-3)^n$

$\frac{2}{1st}, \frac{-6}{}, \frac{18}{3rd}, \frac{-54}{}, \frac{162}{5th}$

$a_3 = -\frac{2}{3}(-3)^3 = -\frac{2}{3}\left(\frac{-27}{1}\right)$

$\frac{54}{3} = 18$

5) Find the 10^{th} term of the sequence: $a_9 = 180,\ r = \frac{1}{3}$

$\frac{540}{8^{th}}, \frac{180}{9^{th}}, \frac{60}{10^{th}}$ $a_{10} = 60$

6) Find the 10^{th} term of the sequence: $a_{12} = -36,\ r = 2$

$\frac{-9}{10^{th}}, \frac{-18}{}, \frac{-36}{12^{th}}$ $\frac{-36}{2} = -18$

$a_{10} = -9$

11-4 Arithmetic Series

of terms

$S_n = \frac{n}{2}(a_1 + a_n)$, n = *which term*, $a_1 = 1^{st}$ *term*, a_n = *last term*

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Tell if it is a *sequence* or *series*. Tell if it is *infinite* or *finite*.

1) $3 + 8 + 13 + 18$

Series

Finite

2) $7, 5, 3, \ldots$

Sequence

Infinite

3) $13 + 12 + 11 + \ldots$

Series

Infinite

4) Write the related series. Evaluate the series: -12, -2, 8, 18

$(-12)+(-2)+(8)+(18)$

$S_4 = \frac{4}{2}(-12 + 18)$

$S_4 = 2(6) = 12$

5) The sequence has 8 terms.

Evaluate the series:

2, 6, 10, … , 30

$S_8 = \frac{8}{2}(2+30)$

$S_8 = 4(32) = 128$

6) Evaluate the series to the 7th term: $a_7 = 3$

24 $21 + 18 + 15 + \ldots$ $d = -3$

$a_n = 24 - 3n$

$a_7 = 24 - 3(7) = 24 - 21 = 3$

$S_7 = \frac{7}{2}(21 + 3)$

$S_7 = \frac{7}{2}\left(\frac{24}{1}\right)$ $S_7 = 84$

11-5 Geometric Series

Finite Series: $S_n = \frac{a_1(1-r^n)}{1-r}$	*Infinite Series*: $S_\infty = \frac{a_1}{1-r}$ $\lvert r \rvert < 1$ Decay

1) Evaluate for the specified number of terms: $a_1 = 1$

$1 + 3 + 9 + \ldots$; $n = 7$

$r = \frac{3}{1} = 3$

$S_7 = \frac{1(1-3^7)}{1-3}$

$S_7 = \frac{1(1-2187)}{-2} = \frac{-2186}{-2}$

$S_7 = 1093$

Which series have a sum?

2) $120 + 60 + 30 + \ldots$ Decay

$r = \frac{60}{120} = \frac{1}{2}$

$r < 1$ yes

3) $5 + 15 + 45 + \ldots$ Growth

$r = \frac{15}{5} = 3$

$r > 1$ NO

4) Evaluate the infinite geometric series:

$3+\frac{1}{3}+\frac{1}{27}+\ldots$

$a_1 = 3$

$r = \frac{1}{3} \div 3$

$r = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$

$S_\infty = \frac{3}{1-\frac{1}{9}} = \frac{3}{\frac{9}{9}-\frac{1}{9}}$

$S_\infty = \frac{3}{\frac{8}{9}} = 3 \div \frac{8}{9} = \frac{3}{1} \cdot \frac{9}{8}$

$S_\infty = \frac{27}{8}$

5) Is the series *arithmetic* or *geometric*? Evaluate for the specified number of terms.

$-3 + 12 - 48 + \ldots ; n = 6$

$r = \frac{12}{-3} = -4$

$S_6 = \frac{-3(1-(-4)^6)}{1-(-4)}$

$S_6 = \frac{-3(1-4096)}{5}$

$S_6 = \frac{-3(-4095)}{5}$

$S_6 = 2457$