4.1 – Antiderivative and Indefinite Integration

F is the antiderivative of f

Antiderivative = Indefinite Integral

\[ \int f'(x)dx = f(x) + C \]

\[ y = f(x) \rightarrow \frac{dy}{dx} = f'(x) \rightarrow dy = f'(x)dx \rightarrow \int dy = \int f'(x)dx \rightarrow y = f(x) \]

Particular Solution – Integrate and then plug in (x,y) to find C.

4.3 – Riemann Sums and Definite Integrals

Riemann Sum: \[ \sum_{i=1}^{n} f(c_i) \Delta x_i \]

If f is continuous and nonnegative on [a,b], then the area of the region bounded by the graph of f, the x-axis, and vertical lines x = a and x = b is given by the definite integral. (Negative area means it is below the x-axis)

Definite Integral: Area \[ \lim_{n \to \infty} \sum_{i=1}^{n} f(c_i) \Delta x_i = \int_{a}^{b} f(x)dx \]

Continuity Implies Integrability: If f is continuous on [a,b] then it is integrable on [a,b].

If f is defined at x = a, then \[ \int_{a}^{a} f(x)dx = 0 . \]

If f is integrable on [a,b], then \[ \int_{a}^{b} f(x)dx = -\int_{b}^{a} f(x)dx . \]

If f is integrable on the three closed intervals determined by a, b, and c, then \[ \int_{a}^{b} f(x)dx = \int_{a}^{c} f(x)dx + \int_{c}^{b} f(x)dx . \]

4.4 – The Fundamental Theorem of Calculus Guidelines p.276

Fundamental Theorem of Calculus: If f is continuous on [a,b] and F is an antiderivative on [a,b], then \[ \int_{a}^{b} f(x)dx = F(b) - F(a) . \]

Mean Value Theorem: If f is continuous on [a,b], then there exists a number c in [a,b] such that \[ \int_{a}^{b} f(x)dx = f(c)(b-a) . \]

Average Value: If f is integrable on [a,b], then the average value of f on the interval is: \[ \frac{1}{b-a} \int_{a}^{b} f(x)dx . \]

Second Fundamental Theorem of Calculus: \[ \frac{d}{dx} \left[ \int_{a}^{x} f(t)dt \right] = f(x) \]

4.5 – Integration by Substitution Guidelines p.292

U-Substitution (Change of Variables): If u = g(x) and du = g'(x)dx , then \[ \int_{a}^{b} f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du . \]

If f is an Even function, then \[ \int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx . \]

If f is an Odd function, then \[ \int_{-a}^{a} f(x)dx = 0 . \]

4.6 – Numerical Integration

Trapezoid Rule: \[ \int_{a}^{b} f(x)dx = \frac{b-a}{n} \left[ f(x_0) + 2f(x_1) + ... + 2f(x_{n-1}) + f(x_n) \right] , n = \text{the number of trapezoids.} \]