7.1 Area of a Region Between Two Curves

$$A=\int_a^b [f(x)-g(x)]\,dx$$

$f(x) \geq g(x)$

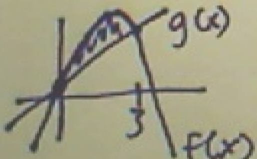
20) Area Between Curves:

$f(x)=-x^2+4x+1$, $g(x)=x+1$

$-x^2+4x+1=x+1$

$0=x^2-3x$ $\quad$ $0=x(x-3)$

$x=0 \;\&\; 3$



$f(1)=-(1)^2+4(1)+1$

$f(1)=4$

$g(1)=1+1=2$

$$\int_0^3 \left[(-x^2+4x+1)-(x+1)\right]dx$$

$$\int_0^3 -x^2+3x\,dx=\left[-\frac{x^3}{3}+\frac{3x^2}{2}\right]_0^3$$

$\left(-\frac{27}{3}+\frac{27}{2}\right)-0$ $\quad$ $\cancel{\frac{27}{6}}$ $\quad$ $-9+\frac{27}{2}$ $\quad$ $-\frac{18}{2}+\frac{27}{2}$ $\quad$ $\frac{9}{2}$

28) Area Between Curves:

$f(y)=y(2-y)$, $g(y)=-y$

$2y-y^2=-y$ $\quad$ $0=y^2-3y$

$f(1)=1(2-1)=1$ $\quad$ $y=3 \;\&\; 0$

$g(1)=-1$

$$\int_0^3 (2y-y^2)-(-y)\,dy$$

$$\int_0^3 -y^2+3y\,dy \quad \frac{9}{2}$$

36) Area Between Curves: $y = x^4 - 2x^2$, $y = 2x^2$

$x^4 - 2x^2 = 2x^2$

$x^4 - 4x^2 = 0$

$x^2(x^2 - 4) = 0$

$x = 0, \pm 2$

$(-1)^4 - 2(-1)^2 = -1 \quad 2(-1)^2 = 2$

$(1)^4 - 2(1)^2 = -1 \quad 2(1)^2 = 2$

$\int_{-2}^{0} (2x^2) - (x^4 - 2x^2)\,dx + \int_{0}^{2} (2x^2) - (x^4 - 2x^2)\,dx$

$\int_{-2}^{0} -x^4 + 4x^2\,dx + \int_{0}^{2} -x^4 + 4x^2\,dx$

$\left[\frac{-x^5}{5} + \frac{4x^3}{3}\right]_{-2}^{0} + \left[\text{'' \quad ''}\right]_{0}^{2}$

$(0) - \left(\frac{+32}{5} - \frac{+32}{3}\right) + \left(\frac{-32}{5} + \frac{32}{3}\right) - (0)$

$\frac{-64}{5} + \frac{64}{3}$

44) Area Between Curves:

$f(x) = \sin x$, $g(x) = \cos 2x$, $-\pi/2 \le x \le \pi/6$

$\sin x = \cos 2x$

$\sin x = 1 - 2\sin^2 x$

$2\sin^2 x + \sin x - 1 = 0$

$(\sin x + 1)(2\sin x - 1) = 0$

$\sin x = -1 \quad \sin x = \frac{1}{2}$

$x = -\frac{\pi}{2} \quad x = \frac{\pi}{6}$

2×1, -2, -1

$(x + \frac{2}{2})(x - \frac{1}{2})$

$(x+1)(2x-1)$

$f(0) = \sin 0 = 0$

$g(0) = \cos(0) = 1$

$\int_{-\pi/2}^{\pi/6} (\cos 2x - \sin x)\,dx$

$\left[\frac{1}{2}\sin 2x + \cos x\right]_{-\pi/2}^{\pi/6}$

$\left(\frac{1}{2}\sin\frac{\pi}{3} + \cos\frac{\pi}{6}\right) - \left(\frac{1}{2}\sin(-\pi) + \cos\left(-\frac{\pi}{2}\right)\right)$

$\left(\frac{1}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) - (0 + 0) = \frac{\sqrt{3}}{4} + \frac{2\sqrt{3}}{4} = \frac{3\sqrt{3}}{4}$

7.2 Volume: The Disk Method

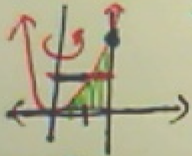
Disk: $V=\pi\int_a^b [R(x)]^2\,dx$

Washer: $V=\pi\int_a^b \left(\overset{\text{larger}}{[R(x)]^2}-\overset{\text{smaller}}{[r(x)]^2}\right)dx$

12) $y=2x^2$, $y=0$, $x=2$

a) y-axis

$y=2(2)^2=8$



$\frac{y}{2}=\frac{2x^2}{2}$ $\quad \sqrt{x^2}=\sqrt{\frac{y}{2}}$

$x=\pm\sqrt{\frac{y}{2}}$

$$\pi\int_0^8 (2)^2-(f(y))^2\,dy$$

$$\pi\int_0^8 4-\left(\frac{y}{2}\right)dy$$

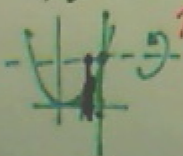
b) x-axis

$$\pi\int_0^2 (2x^2)^2\,dx$$

$$\pi\int_0^2 4x^4\,dx$$

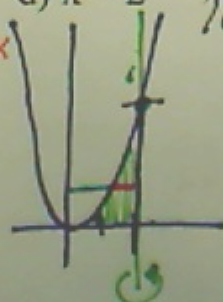
$$\pi\left[\frac{4x^5}{5}\right]_0^2$$

c) y = 8



$$\pi\int_0^2 (8)^2-(8-2x^2)^2\,dx$$

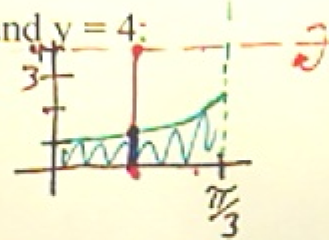
d) x = 2



$$\pi\int_0^8 \left(2-\sqrt{\frac{y}{2}}\right)^2 dy$$

18) Volume by revolving around $y = 4$:

$y = \sec x\ ,\ y = 0\ ,\ 0 \le x \le \pi/3$



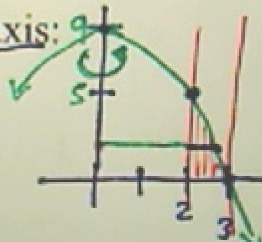
$\pi\int_0^{\pi/3} (\ 4\ )^2 - (4-\sec x)^2\, dx$

$\pi\int_0^{\pi/3} 16 - (16 - 8\sec x + \sec^2 x)\, dx$

$\pi\int_0^{\pi/3} 8\sec x - \sec^2 x\, dx = \underline{\qquad}\ u^3$

32) Volume by revolving around y-axis:

$y = 9 - x^2\ ,\ y = 0\ ,\ x = 2\ ,\ x = 3$



$V = \pi\int_0^5 (\underbrace{\sqrt{9-y}}_{\text{outer}})^2 - (\underbrace{\ 2\ }_{\text{inner}})^2 dy$

$y = 9 - x^2$

$\sqrt{x^2} = \sqrt{9-y}$

$x = \pm\sqrt{9-y}$

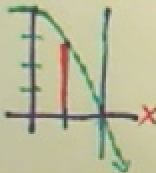
$V = \pi\int_0^5 (9-y) - (4)\, dy = \pi\int_0^5 5 - y\, dy = \pi\left[5y - \frac{y^2}{2}\right]_0^5 \rightarrow$

$\rightarrow \pi\left[\left(25 - \frac{25}{2}\right) - (0)\right] = \pi\left(\frac{25}{2}\right) \quad V = \frac{25\pi}{2}$

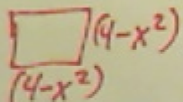
1) Volume by creating cross sections perpendicular to the x-axis:

$y = 4 - x^2$, $y = 0$, $x = 0$, $x = 2$

a) Squares:

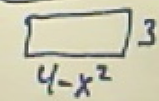


$$\int_0^2 (4-x^2)^2\,dx$$



b) Rectangles: (h=3)

$$\int_0^2 3(4-x^2)\,dx$$



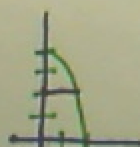
2) Volume by creating cross sections perpendicular to the y-axis:

$y = 4 - x^2$, $y = 0$, $x = 0$, $x = 2$

a) Semicircles:

$y = 4 - x^2$

$x = \sqrt{-y+4}$

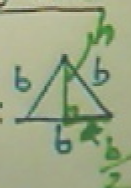


$r = \frac{\sqrt{4-y}}{2}$

$$A = \frac{\pi\left(\frac{\sqrt{4-y}}{2}\right)^2}{2}$$

$$\frac{1}{8}\int_0^4 \pi(4-y)\,dy$$

b) Equilateral Triangles:



$h^2 + \left(\frac{b}{2}\right)^2 = b^2$

$h = \sqrt{b^2 - \frac{b^2}{4}}$

$A = \frac{b}{2}\left(\sqrt{\frac{3b^2}{4}}\right)$

$b = \sqrt{4-y}$

$$\int_{\cancel{0}\,2}^{\cancel{4}\,0} \frac{b}{2}\left(\sqrt{\frac{3b^2}{4}}\right)db$$